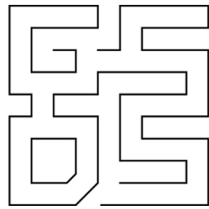
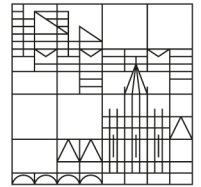


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No. 2016-03

Sources of German Income Inequality across Time and Space

Franziska K. Deutschmann

August 2016

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ISSN: 2365-4120

August 2016

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Sources of German Income Inequality across Time and Space

Franziska K. Deutschmann*

Income inequality rose in Germany since the 1970s. To quantify the impact of socio-economic trends on inequality, I construct counterfactual distributions of net household income with rich German data. The procedure controls for marital sorting in education and allows for indirect influences, such as the influence of education on employment. I find that the prevalence of singlehood can account for the observed increase in inequality since the 1970s to a vast extent. The inequality increase is also associated with a change of employment among males and single females. Compared to West Germany, in 2011, the stronger labor attachment of East German married females combined with the high East German unemployment produce more income inequality. East-West differences in children per household also boost East German inequality. East German education works against it. I find no evidence that positive assortative mating in education or the aging society augment income inequality significantly.

JEL classification: D31, E25, I24, J11, J12, J21.

Keywords: inequality, demography, household structure, assortative mating, education, employment, Germany

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1. Introduction

Similar to most industrialized countries, the net income of German households has become more unequal since the 1970s.¹ Recent literature emphasizes, among others (e.g. changes in the tax and transfer system, capital income, or workplace heterogeneity; see Card, Heining & Kline, 2013, Schmid & Stein, 2013, Biewen, 2012, and Rehm, Schmid & Wang, 2014), societal trends as determinants for this rise in income inequality (Burtless, 2011, Greenwood, Guner, Kocharkov & Santos, 2014, Gregorio & Lee, 2002, Kollmeyer, 2013, Martin, 2006, Peichl, Pestel & Schneider, 2012, Reed & Cancian, 2012). Such societal trends are changes in the demography, education, preferences for marital sorting, and the labor force participation. Take, for example, the rising share of single parents. Single parents are more likely to live in poverty than couples without children (see Buscher & Parys, 2006). When the number of single parents increases, the share of poor households also rises, and income becomes more unequal across the households. From a politics perspective, before counteracting these social trends or their effects on income inequality, it is important to understand whether and by how much each specific socio-economic trend influences income inequality.

The first contribution of my paper is to disentangle the contribution of several societal trends on the German rise in income inequality. Why do I study income inequality for Germany? Germany has one advantage over other industrialized countries. Although facing the same political system for 20 years, in 2011, East and West Germany still differ substantially in their socio-economic characteristics.² These differences allow a cross-regional comparison of the societal impacts on income inequality. In the second part of the paper, I observe how these East-West-differences in socio-economic household characteristics influence income inequality in East and West Germany.³

I apply the idea of the reweighting procedure by Di Nardo, Fortin & Lemieux (1996) in a

¹For details on the rise in income inequality in industrialized countries, see, for example, Gottschalk & Smeeding (2000), and OECD (2008, 2011). For details on the rise in German income inequality, see, for instance, Hauser & Becker (1998), Becker (2012), Biewen (2000), and Bach, Adam, Niehues, Schröder, Frey, Schaltegger, Berthold & Gründler (2014).

²For a documentation on the distinct age distributions in East and West Germany, see Grünheid (2009) and Brenke (2014). For the evolution of East and West German household size, see Grünheid (2009) and Ebert & Fuchs (2012). For development of male and females education as well as assortative mating in both German regions, see Grave & Schmidt (2012), Wirth (1996) and Wirth (2014). For a documentation on German employment, see, for example, Ebert & Fuchs (2012) or Holst & Wieber (2014).

³I am not interested in the reason why these differences still exist or whether people with specific characteristics migrated to the East or to the West. If these East-West-movers (West-East movers) face the same conditional income distribution as their West (East) German peers, it is irrelevant whether parts of the West (East) German workforce is originally from the East (West).

non-parametric fashion and extend it by marital sorting algorithms. I construct counterfactual income distributions by adopting the composition of the population with respect to a specific attribute from another year or region. Attributes are the households age, the cohabitation status, the number of children, the adults' gender, their educational degree and their working status. The procedure indirectly takes the tax and transfer system into account as the adult-equivalent net income is held constant conditional on the household attributes. Moreover, unlike most decomposition approaches, the sequential approach proposed by Di Nardo et al. (1996) allows for holding some household characteristics constant while taking indirect effects into account. For example, when adjusting the composition of the population with respect to education, the populations' age is fixed, but a change in female education may affect the overall number of children and female employment. As a consequence, the reweighting procedure depends on causality assumptions, and like other decomposition techniques (e.g. the Oaxaca-Blinder and the RIF decomposition), it neglects general equilibrium effects.

Within the reweighting procedure, marital sorting can easily be controlled for. I control for marital sorting in formal education in three different ways: fixing marital sorting while adjusting gender-specific education (algorithm by Sinkhorn & Knopp, 1967), randomizing marital sorting while fixing gender-specific education, and perfectly sorting males and females regarding education while fixing their gender-specific education (algorithm by Gale & Shapley, 1962).⁴ To my knowledge, this is the first study on German data combining a reweighting procedure and sorting algorithms.

I use data from the German microcensus. The microcensus is several times larger than the the German socio-economic panel (GSOEP), which is usually used in empirical studies.⁵ The rich microcensus allows for a finer categorization of household attributes. With this finer categorization, the present study provides detailed information on the sources of income inequality. Moreover, all relevant information are available in the German microcensus since 1976. Because the time span surveyed is longer than in the GSOEP, I may observe more demographic and educational variation between the first and the last survey year. This larger variation could provide results which have not been found significant in previous studies.

⁴Controlling for changes in marital sorting is necessary, as an increase in female or male education may drive the growing likelihood that males and females with the same educational degree marry each other or may offset this trend of increased positive assortative mating.

⁵See, for example, Biewen & Juhasz (2012), Peichl et al. (2012), Pestel (2016), Rehm et al. (2014), Schmid & Stein (2013). One exception is the study by Buscher & Parys (2006) on poverty in East and West German households, but they only consider microcensus data from 1996 to 2002.

For the comparison *across time*, I find that changes in the household size, education, and employment account for the rise in West German income inequality to a significant extent. Thereby, the rising prevalence of singlehood plays the most important role. Exercises on the effects of employment show that the working status of males and single females does, but the working status of married females does not contribute to the rise in income inequality. These results augment the findings by Biewen & Juhasz (2011, 2012) and Peichl, Pestel & Schneider (2011), and Peichl et al. (2012) which reveal the importance of declining employment and smaller households for the rising German income inequality. Besides, in line with the findings on earnings inequality by Pestel (2016), I find that marital sorting in education has no potential to influence income inequality in West Germany. Faik (2012) forecasts that the aging society will cause more income inequality in the future. Although the workforce exhibits the aging trend already since 1976, I cannot find a significant impact of changes in the household's age on income inequality.

By comparing the influence of household characteristics *across space*, I find that the high unemployment among East German males and single females makes income more unequally distributed. Fewer children in couple households combined with a stronger presence of children in single households, like in East Germany, also increases the income gap. These findings are in line with the studies on the East German inequality increase by Biewen (2001) and Peichl et al. (2012). Their studies link the growth in the income gap in the early 1990s and 2000s to rising unemployment, declining female labor force participation, and smaller households. Moreover, my results suggest that both rather equal education across East German households and smaller variations in the conditional income distributions offset the inequality generated by employment and household size. Like for West Germany, for East Germany, I cannot confirm the hypothesis that marital sorting in education promotes income inequality. My results indicate that the German tax and transfer system counteracts the disequalizing effect which arises from marital sorting in education (see Fuchs-Schündeln, Krueger & Sommer, 2010, Pestel, 2016) or marital sorting occurs in terms of something else than formal education, e.g. in terms of occupation or industry.

In the following section, I briefly discuss methods for studying income inequality. In Section 3, I inform about the data set used and describe the socio-economic background of German households. The analytic framework is introduced in Section 4. Section 5 presents the results of the counterfactual exercises. In the final section, I discuss the main results and conclude.

2. A Brief Review of Methodical Approaches

In order to understand differences in inequality an extensive spectrum of methods has been used in previous studies.⁶ A range of descriptive papers exist. They link changes in policy and demography to shifts in economic inequality (see e.g. Gottschalk & Smeeding, 2000, Weick, 1995). However, these studies cannot provide a quantification of how much a specific demographic or political factor contributes to inequality.

Another branch of the literature utilizes regression models, in which the Gini coefficient is regressed on household or regional characteristics (e.g. Gregorio & Lee, 2002, Kollmeyer, 2013, Li, Squire & Zou, 1998). As the Gini coefficient is a non-linear and bounded measure, effects quantified by simple regression analysis provide only limited information. The recentered influence function (RIF) approach by Firpo, Fortin & Lemieux (2009) does not suffer from this caveat because it regresses the RIF of the Gini coefficient, and not the Gini coefficient itself, on the covariates. However, Rothe (2015) argues that the RIF method is not always applicable, as it relies on strong assumptions. If the assumptions are not fulfilled, the RIF method can lead to arbitrary larger errors. For example, when it comes to the distributions of formal education in East and West Germany, the assumptions are not satisfied: The distributions of education are no location-shifted versions of each other. The variance in formal education is larger in the West whereas education is on average similar in both regions. The RIF method is not applicable.

For a better understanding of how inequality is affected by changes in the population composition or the income structure, the goal of many studies is to decompose changes in inequality. Thereby, the method of how to decompose generalized entropy indices or other inequality measures may vary substantially.⁷ A path-dependent decomposition has been proposed by Di Nardo et al. (1996). They analyze the impact of the labor market and institutional factors on changes in wage inequality. Changes in the population characteristics and in the conditional wage distribution are both taken into account. This semi-parametric approach has been adopted for the analysis of income inequality growth

⁶For a discussion of popular empirical strategies to determine the underlying sources of inequality, see Fortin, Lemieux & Firpo (2011).

⁷Three examples are: Inequality change is split into the effect of income growth and reranking (e.g. Jenkins & van Kerm, 2006); Income is decomposed by its sources and their contribution to overall income inequality is quantified (e.g. Aslaksen, Wennemo & Aaberge, 2005, Becker, 2000, Cancian & Reed, 1998, Karoly & Burtless, 1995, Lerman & Yitzhaki, 1985); The population is separated into pairwise disjoint subgroups, and within-group as well as between-group effects, and in some studies even group size effects, are measured (e.g. Bönke, Corneo & Lüthen, 2015, Peichl et al., 2011, 2012, Schwarze, 1996, Shorrocks, 1984, Western, Percheski & Bloome, 2008).

in the US (Daly & Valletta, 2006), Norway (Eika, Mogstad & Zafar, 2014), New Zealand (Hyslop & Maré, 2005), Germany (Biewen & Juhasz, 2011, 2012, Peichl et al., 2012), and East Germany (Biewen, 2001). I also apply the idea introduced by Di Nardo et al., but incorporate marital sorting algorithms without estimating Kernel densities.⁸

3. Data and Descriptive Statistics

The German microcensus is the largest representative household survey in Europe. It covers 1 percent of the German population. I utilize the scientific-use files of the microcensus from 1976 to 2011.⁹ The first survey year in which formal educational levels are reported is 1976. Since 1991 the survey also covers East German households. Whereas in the 1970s cohabiting males and females used to be married, non-married couples are more common nowadays. In the microcensus of 1976, it was not considered that non-married males and females run a household together, and so I abstract from such households in 1976. Only since 1996, households which are run by a non-married couple are reported. As I do not focus on tax effects of marriages, all married and cohabiting couples refer to *married* households in 2011 and the term *married* is used as a synonym for not being single.¹⁰

For the household income, I apply the reported monthly net income.¹¹ It is divided by the OECD-modified scale to receive an adult-equivalent income for each household. The scale assigns a value of 1 to single households, 1.5 to couple households and, additionally, 0.3 for each child.

Households with more than two adults are not considered. Households including adult children or same-sex couples are excluded as their sample size is too small for the analysis.¹² Because my study does not focus on evaluating the income transfers to pensioners, I restrict

⁸The German microcensus observes income within given intervals, but the data is rich so that I directly use the categorized income distribution and abstract from estimating kernel densities. This simplification does not influence the findings. For the West German inequality increase, it provides results in line with Biewen & Juhasz (2012) and Peichl et al. (2012). Both are reweighting studies which use the Kernel density estimates to smooth the income distribution as it is introduced by Di Nardo et al. (1996).

⁹The scientific-use files encompasses 70 percent of the data, i.e. 0.7 percent of all German households.

¹⁰Besides, social norms change. 27% (61%) of all firstborn West (East) German babies are born to an unmarried mother (Pötzsch, 2012). Hence, there is no reason to restrict the analysis to married couples.

¹¹Appendix Section D documents how the reported income categories from the microcensus are used to derive income levels.

¹²Most children older than 19 who live with their parents are either students at a university, or they finished education and work. They may work part-time, full-time, or not at all. Covering all these different types of adult children in the analysis results in an inflated categorization. Same-sex couples are not surveyed before 1996, and in the recent surveys there are too few same-sex couples to categorize them by age, gender, and their educational degrees simultaneously.

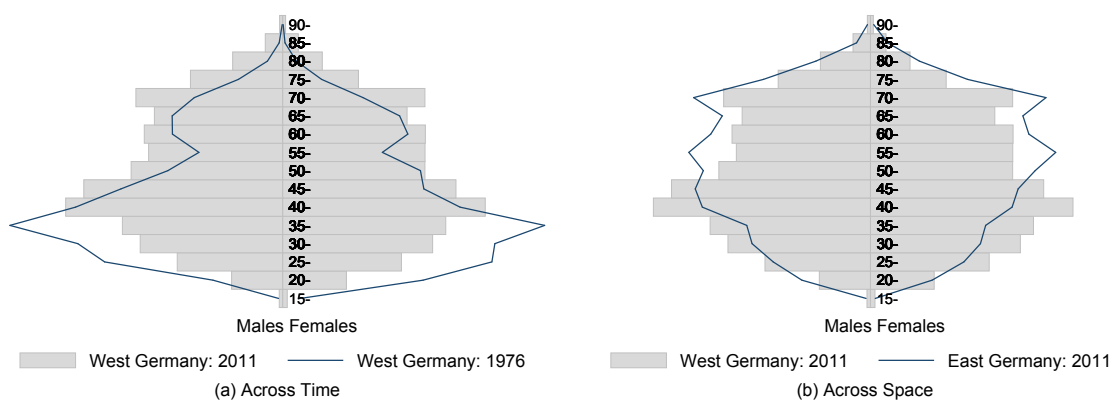
the data to households with a household head of working age; that is, the male adult or, if absent, the single female adult is younger than 60 and at least 25 years old. For a useful categorization of households by age, I abstract from couples whose age difference is larger than 10 years. Thereby, the sample diminishes by less than 5%. In the end, the sample consists of approximately 137,000 households: 55,109 for West Germany in 1976, 67,749 for West Germany in 2011, and 14,055 for East Germany in 2011.¹³

How do West German households differ across time and compared to East German households? To answer this question, I present and discuss the age distribution, the cohabitation, education, and employment situation of German households in 1976 and 2011 in the following.

Aging Society

It is well-known that the German population is aging. This trend of an aging population is present in both parts of Germany. Since the 1990s, it is even more pronounced in the East (see Grünheid, 2009). For my sample, Figure 1 illustrates the age distribution of West German men and women in 2011 compared to 1976 and compared to East Germany.¹⁴ The left-hand panel shows that, in West Germany in 2011, the fraction of people who are older than 40 is larger than in 1976. The right-hand panel of Figure 1 shows that, in 2011, the share of people older than 50 is even larger in East Germany than in West Germany. For the share of younger people, the opposite is observed.

Figure 1: Normalized Age Distribution (15 - 95 Years)



¹³Berlin is excluded from the sample.

¹⁴The figure represents the sample before restricting the age of the household head, but after introducing all other restrictions mentioned.

Singlehood and Fertility

In Germany, household size declines steadily since the 1970s. There are two main reasons for this trend: the rise in singlehood and the decline in fertility. Ebert & Fuchs (2012) document this rise in singlehood and the decline in the households with children for both East and West Germany. They also show that, since 2002, the prevalence of singlehood and households without children are more pronounced in the East than in the West. In the West, the share of couples with children has decreased, while the stronger prevalence of singlehood lead to a rising share of singles with children. This increase in single parents has occurred, although it has become less common among singles to have children in the households (see Table 1). In East Germany in 2011, compared to West Germany, the share of single parents is larger because single households are more prevalent and it is more common among singles to have children.

Table 1: Household Composition

Region & year	Couple households	Single households	Average number of children in households
West 1976	84.15 %	15.85 %	1.24
West 2011	49.32 %	50.68 %	0.66
East 2011	45.60 %	54.40 %	0.45

Region & year	Couple households with children	Single households with children	Childlessness rate among singles*
West 1976	62.37 %	3.22 %	79.70 %
West 2011	31.92 %	6.40 %	87.37 %
East 2011	21.23 %	7.96 %	85.38 %

Notes: Households of workforce age, children under 20. * *Childless* refers to households with no children (younger than 20 years old) living with them; the household members may have children who live outside the household.

Buscher & Parys (2006) have shown that the risk of poverty is larger for single than for couple households and increases in the number of children living in the household. Hence, if there are more singles, the share of poor households grows and inequality rises. If single (couple) parents tend to have more children or the share of households with children increases among the singles (couples), income inequality may also grow. However, if the share of (couple) households with children increases, (couple) households may also become more alike; the inequality gap may decline.

Education and Marital Sorting

In Germany, formal education can be classified in three categories: school education, vocational training (apprenticeship), or university (college) education. People with a vocational training have a decent education, better than a school degree, but, in general, it is worth less than a university degree. As Table 2 shows, in 1976, formal education is low in West Germany. Males are on average better educated than females.¹⁵ In 2011, West Germans are on average better educated, but also more unequal regarding their educational degrees. The gender-differences in education have declined. In East Germany, women and men differ even less in their education. Moreover, the East German population is rather equally educated, as more than three-quarter have an apprenticeship degree.

If both partners have the same level of education, a couple is called *educational homogamous*. In 2011, the predominance of apprenticeships in East Germany creates strong educational homogamy (see Table 2). In West Germany, educational homogamy is still less pronounced than in the East, although it has increased since the 1970s (Grave & Schmidt, 2012, Wirth, 1996, 2014).

Table 2: Education

(a) Educational Attainment, in Percent						
Data	Females			Males		
	University	Apprenticesh.	School	University	Apprenticesh.	School
West 1976	3.56	50.28	46.15	9.11	71.15	19.74
West 2011	20.02	66.07	13.91	23.82	65.02	11.15
East 2011	16.76	77.87	5.37	16.09	78.75	5.16

(b) Educational Homogamy		
Data	Fraction of homogamous couples	Fraction of homogamous couples relative to random matching
West 1976	59.50 %	+ 31 %
West 2011	69.76 %	+ 38 %
East 2011	80.76 %	+ 23 %

¹⁵For an overview of the evolution of education within marriages from 1976 to 2005, see Grave & Schmidt (2012).

Educational homogamy is not only driven by gender-specific education, but also by *marital sorting*. In particular, in West Germany in 2011, marital sorting plays an essential role. If West German males and females are matched randomly, 49 percent of the couples would have the same educational degree. However, the observed share of homogamous couples is larger by 38 percent (see Table 2). Compared to West Germany in 2011, marital sorting is less important for educational homogamy in 1976 and plays an even subordinate role for the East.

The idea of how education affects income inequality is straightforward. The skill-premium makes income strongly correlated with education. Low inequality in education is associated with low income inequality.¹⁶ For West Germany, this means that the education from the 1970s may equalize income across households because the expansion of education made West Germans more dispersed in their education.

The idea of how marital sorting affects income inequality is equally straightforward. When marriages occur between low-educated and well-educated people, their different income levels balance out within the household; but marriages between low-educated (highly educated) people produce relatively poor (rich) households. If the number of marriages between high-educated (low-educated) people is increasing, the number of relatively rich (poor) households would rise, as well as income inequality across households.

Employment

Figure 2 illustrates the employment of single and married men and women in Germany across time and space. Here, adult household members are categorized as either jobless, full-time working or part-time (between 0 and 35 hours) working. Thereby, unemployed people are not distinguished from people who are not in the labor force. The terms *jobless*, *unemployed* and *not in the labor force* are used interchangeably.

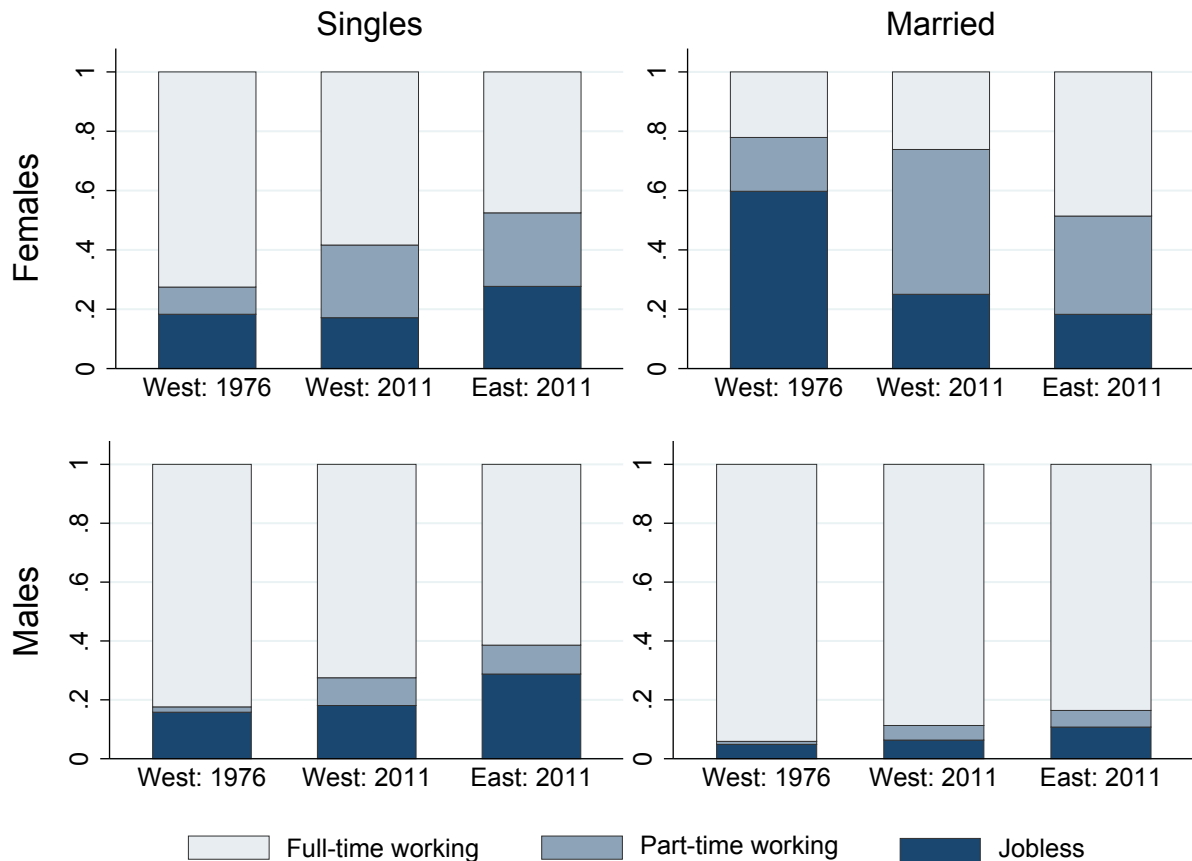
Between 1976 and 2011, part-time employment gained importance in West Germany. In 2011, married females are less often jobless in favor of part-time work. Contrary, single females work fewer hours in favor of part-time work. Among males, part-time work has

¹⁶However, this hypothesis underlies the assumption that employment is independent of educational levels. In Germany, the fraction of working females is largest among the highly educated (see Figure A.1 in the Appendix). Suppose the following extreme case: Most females are low-educated, and only the highly educated females are working. In the presence of positive assortative mating, highly educated females are more likely to marry highly educated males (see Table B.1 for the contingency tables across time and region), and both together are likely to face a household income far above median levels. Then, a rising share of highly educated females, and not the dispersion in education per se, would boost income inequality across households.

also increased since 1976. However, the joblessness rate among males has increased as well. This rise in jobless and part-time working males goes along with a decline in full-time working males which produces more inequality in the employment of West German men.

Compared to the development of the West German employment until 2011, the East German employment in 2011 can be viewed as going one step further in time. Males and single females are more often jobless than in the West; their employment is lower and more dispersed. Moreover, compared to West Germany in 2011, married females are less often jobless and less often part-time working in favor of full-time work.

Figure 2: Employment



When the conditional employment of the household members changes, the same demographic pattern may generate more or less income inequality. Through the impact of labor income, low inequality in the households' employment may produce relatively low income inequality. Moreover, when single females work more hours, their labor income is lifted,

and the income gap to single male households as well as to couple households shrinks. Inequality in adult-equivalent income diminishes.

The impact of a rise in married females' employment is ambiguous. On the one hand, employment of married females may offset a disadvantage in the husbands' income. On the other hand, employment of married females' can amplify the income inequality arising by the husbands.¹⁷

4. Analytical Framework

In order to analyze the impact of several socio-economic factors on income inequality, I use a non-parametric version of the reweighting method by Di Nardo et al. (1996) and extend it by sorting algorithms. The framework is introduced in three stages: First, households are categorized by specific characteristics like age, size, education, etc. Then, the share of households with a specific characteristic is adjusted to a counterfactual scenario. This adjustment is, for example, a decrease in the share of single households to the levels of 1976. By adjusting the size of specific subgroups of the population, the overall income distribution in the population changes. Finally, the resulting change in income inequality is evaluated.

4.1. Household Categorization

For the analysis, weighted households are categorized by the age (H) of the household head, marital or cohabitation status (C), gender (G), the number of children under 20 years old (K), the adults' educational levels (E), and the adults' working status (W).¹⁸ Households which cannot be categorized are excluded from the sample. See Table 3 for a list of the categories and their characteristics.

Household characteristics are summarized in a household type $x = (x_W, x_K, x_E, x_G, x_C, x_H)$. The set of all possible household types is denoted by $X = W \times K \times E \times G \times C \times H$. These are 1584 potential household types, out of which no more than 1208 are surveyed in the

¹⁷See Lam (1997) for details on the correlation of the income of husbands' and wives' and their impact on income inequality across households. If, for example, the variance of the wives' and the husbands' income is the same ($\sigma_W = \sigma_H$), the average income of the wives is smaller than that of the husbands ($\mu_W < \mu_H$), and marital sorting in terms of income occurs ($\rho > 0$), then the wives' average income amplifies inequality across the households. This amplifying effect becomes particularly apparent if ρ is large, i.e., if educational and employment homogamy are strong.

¹⁸Note that every household possessing the gender *male* & *female* belongs to the category *couple*, and vice versa.

Table 3: Categorization of Households

Abbreviation	Description
H	Age of the household head: 25-34, 35-44, 45-54, 55-59
C	Cohabitation status: single, couple household
G	Gender: male & female, female, male
E	Highest educational degree of the adults in the household $= E^m \times E^f$ if couple household $= E^m$ if single male household $= E^f$ if single female household
E^f, E^m	Educational degree by gender: university, apprenticeship, school
K	Children younger than 20 in the household: 0, 1, 2, 3 or more
W	Working status of the adults in the household $= W^m \times W^f$ if couple household $= W^m$ if single male household $= W^f$ if single female household
W^f, W^m	Working status by gender: full-time working (≥ 35 hours), part-time working (> 0 hours, < 35 hours), jobless

microcensus. For instance, there are no 55 to 59 year old, highly educated, but jobless single males with three or more children in the household.

4.2. Counterfactual Income Distributions

For each year and region, the income distribution function of the whole population is denoted by F_Y^i . The i stands for the year and the region: either West Germany in 1976, West Germany in 2011, or East Germany in 2011. In the specific year and region, the income for households of type x is distributed according to the density $f_{Y|X}^i(\cdot, x)$. The share of households who belong to a specific household type x is given by $\mathbb{P}_X^i(x)$. To construct a counterfactual income distribution function F_Y for year and region i , the income densities $f_{Y|X}^i(\cdot, x)$ conditional on household types $x \in X$ are weighted by counterfactual shares of the household types $\mathbb{P}_X(x)$:

$$F_Y(I) = \int_0^I \sum_x f_{Y|X}^i(y, x) \mathbb{P}_X(x) dy. \quad (1)$$

In order to construct the counterfactual shares of household types $\mathbb{P}_X(x)$, household char-

acteristics X are decomposed into 3 parts:

X_{fix} : regarding to these characteristics the population is held constant

X_{ex} : characteristics which are changed conditionally on X_{fix}

X_{ind} : the rest.¹⁹ Regarding the characteristics X_{ind} , the population may change due to the conditionality on $X_{ex} \times X_{fix}$.

For example, when I consider the impact of the overall increase in education from 1976 to 2011, I hold the age distribution and the fraction of single and couple households constant ($X_{fix} = G \times C \times H$); I change the fraction of well-educated and less-educated households conditionally on age and gender ($X_{ex} = E$); the employment and the number of children in the households are fixed conditionally on the households' age, cohabitation status, gender, and education ($X_{ind} = W \times K$). Then, the adjustment of male and female education might have an effect on overall employment as well as on the overall number of children.

In all exercises, I assume that the distribution of the household's age is independent of any other household characteristic. It is well-known that females get older than men, and males are older when they start cohabiting. Therefore, I condition the share of couple, single male, and single female households on the household's age. By construction, the either two-dimensional or one-dimensional educational level of the household depends on the cohabitation status and the household members' gender. Because I condition the adults' gender on the household's age, education is also conditioned on age. In Germany, the educational degree is to a vast extent predetermined by the school track chosen at the age of ten. Fertility decisions are made later in live; therefore, I condition the number of children on the household's education. I assume that the number of children living in the household has an effect on the working status of the adult household members. Moreover, I assume that married females adjust their working status depending on their husbands' employment.

For the sake of simplicity, in the following A and B describe two distinct subsamples which are categorized according to X . A and B may stand for West Germany in 1976, West Germany in 2011, or East Germany in 2011.

How would income inequality change if households in sample A (e.g. West Germany in 2011) are characterized in terms of their characteristic X_{ex} as in sample B (e.g. West

¹⁹By definition, everything which is not explicitly controlled for in either X_{ex} or X_{fix} is held constant conditional on $X_{ex} \times X_{fix}$, because $F_Y(I) = \int_0^I \sum_x f_{Y|X}(y, x) \mathbb{P}_X(x) dy = \int_0^I \sum_{x \in X} \sum_{z \in Z} f_{Y|Z, X}(y, z, x) \mathbb{P}_{Z|X}(z, x) \mathbb{P}_X(x) dy$ and $\mathbb{P}_{Z|X} \mathbb{P}_X = \mathbb{P}_{Z|X} \mathbb{P}_{(X_{ind}|X_{ex}, X_{fix})} \mathbb{P}_{X_{ex}|X_{fix}} \mathbb{P}_{X_{fix}} = \mathbb{P}_{(Z, X_{ind}|X_{ex}, X_{fix})} \mathbb{P}_{X_{ex}|X_{fix}} \mathbb{P}_{X_{fix}}$. The known framework follows with $\tilde{X}_{ind} := Z \times X_{ind}$.

Germany in 1976)? The corresponding counterfactual shares of household types $\mathbb{P}_X(x)$ are implemented by exchanging the conditional distribution $\mathbb{P}_{X_{ex}|X_{fix}}^A$ by $\mathbb{P}_{X_{ex}|X_{fix}}^B$ in the Bayes' formula, that is

$$\mathbb{P}_X = \mathbb{P}_{(X_{ind}, X_{ex}, X_{fix})} = \mathbb{P}_{(X_{ind}|X_{ex}, X_{fix})}^A \mathbb{P}_{X_{ex}|X_{fix}}^B \mathbb{P}_{X_{fix}}^A. \quad (2)$$

Given the counterfactual shares of household types as constructed in equation (2), the following assumption ensures that the counterfactual income distribution function F_Y in equation (1) is well-defined.

Assumption 1 For all $x \in X$: $\mathbb{P}_X^A(x) = 0$ if and only if $\mathbb{P}_X^B(x) = 0$.

The assumption is satisfied when each household type which is represented in sample A also occurs in sample B , and vice versa. In order to generate well-defined counterfactual income distribution functions for the exercises across time (space), I include only those household types which are present in West Germany in both years 1976 and 2011 (in 2011 in both regions East and West Germany).²⁰ For a detailed description for each exercise, see the Appendix, Section C.

Marital Sorting in Education

When a counterfactual exercise on education is conducted, educational sorting in marriages need to be fixed. Adjusting the education within the household's without fixing marital sorting provides no clear results on whether changes in gender-specific education or changes in marital sorting affect income inequality. If X_{fix} characterizes married households of specific age groups, $\mathbb{P}_{E|X_{fix}}^i = \mathbb{P}_{(E^m, E^f)|X_{fix}}^i$ represents contingency tables of male and female educational levels. In the exercises on education, these contingency tables are adjusted for every age group. Similar to Greenwood et al. (2014), I apply the Sinkhorn-Knopp algorithm (Sinkhorn & Knopp, 1967) to fix the mating scheme. This mechanism fixes marital sorting in terms of the local log odds ratio in the contingency tables.²¹ The local log odds ratio is

²⁰Omitting all these households decreases the number of observations by less than 4% (see the Appendix, Table B.3). It does not affect the inequality across households (see the Appendix, Table B.4). For generate well-defined counterfactual income distribution functions, a restriction of the subsample is not necessary in the exercises on age, singlehood, and education. I constructed these exercises also before excluding the household types which do not exist in the other year or region; the findings do not change.

²¹Fixing another measure of assortative mating may provide different results, because two different measures of assortative mating are generally not equivalent; so, the share of homogamous couples relative to random matching varies, although the local log odds ratios are fixed. For an overview of measures for association analysis, see Tan, Kumar & Srivastava (2004).

a typical measure for assortative mating (see, for example, Siow, 2015). It is defined as

$$\log \left(\frac{\mathbb{P}^i(E_k^m, E_l^f) \mathbb{P}^i(E_{k+1}^m, E_{l+1}^f)}{\mathbb{P}^i(E_{k+1}^m, E_l^f) \mathbb{P}^i(E_k^m, E_{l+1}^f)} \right),$$

where $k, l \in \{1, 2\}$, and $E_1^m = E_1^f = \textit{university}$, $E_2^m = E_2^f = \textit{apprenticeship}$, and $E_3^m = E_3^f = \textit{school}$. With the 3-by-3 contingency table of male and female education levels, a 2-by-2 matrix of local log odds ratios results. This matrix can be seen as a generalization of the odds ratio, which is only defined for 2-by-2 contingency tables.²² An advantage of the local log odds ratio is its scale-invariance. The Sinkhorn-Knopp algorithm exploits the scale-invariance by fixing the sorting pattern when adjusting the marginal distributions of male and female education.²³

How would income inequality change if, in sample A , marital sorting across couple households is random, perfect, or the same as in sample B ? I conduct several counterfactual exercises on marital sorting. In the first exercise, I test whether random matching would diminish income inequality. In the second exercise, I assume an educational sorting pattern from another year or region B and test whether marital sorting differs so much that it has a significant impact on income inequality. Finally, I test whether perfect positive assortative matching, in the sense of the Gale-Shapley matching algorithm (Gale & Shapley, 1962), would increase income inequality significantly.

The counterfactual exercises are implemented by adjusting the conditional distribution of the households' education $\mathbb{P}_{E|X_{fix}}^A$ in the formula

$$\mathbb{P}_X^A = \mathbb{P}_{(X_{ind}, E, X_{fix})}^A = \mathbb{P}_{(X_{ind}, |E, X_{fix})}^A \mathbb{P}_{E|X_{fix}}^A \mathbb{P}_{X_{fix}}^A. \quad (3)$$

If X_{fix} characterizes singles, $\mathbb{P}_{E|X_{fix}}$ equals $\mathbb{P}_{E|X_{fix}}^A$. Otherwise, matching algorithms are used to create counterfactual contingency tables $\mathbb{P}_{E|X_{fix}}$ for $\mathbb{P}_{E|X_{fix}}^A$ in equation (3). Thereby, the marginal distribution of male and female education, $\mathbb{P}_{E^m|X_{fix}}^A$ and $\mathbb{P}_{E^f|X_{fix}}^A$, respectively, remain unchanged. For the exercise on random sorting, the entries of the contingency tables

²²However, this matrix of local log odds ratios is not the generalized odds ratio as defined by Agresti (1980). (The generalized odds ratio can be traced back to the association measure of Goodman and Kruskal (see Goodman & Kruskal, 1954) which is a non-scale-invariant generalization of Yules' Q.)

²³The convergence of the Sinkhorn-Knopp algorithm is only ensured, if the contingency tables have no zero entries. With the given data set, there exists only one case in which a contingency table has a zero entry. However, the Sinkhorn-Knopp algorithm still works.

are replaced by

$$\mathbb{P}_{E|X_{fix}}(x_E, x_{fix}) = \mathbb{P}_{(E^m, E^f)|X_{fix}}(x_{E^m}, x_{E^f}, x_{fix}) = \mathbb{P}_{E^m|X_{fix}}^A(x_{E^m}, x_{fix}) \mathbb{P}_{E^f|X_{fix}}^A(x_{E^f}, x_{fix}).$$

In order to adopt the educational sorting pattern from a different year or region B , the contingency tables $\mathbb{P}_{E|X_{fix}}^B$ are transformed with the Sinkhorn-Knopp algorithm. The sorting pattern of B is fixed, but male and female marginal conditional education form A is incorporated. The transformed contingency tables replace $\mathbb{P}_{E|X_{fix}}^A$ in equation (3). For the exercise on perfect sorting, the Gale-Shapley algorithm is applied on $\mathbb{P}_{E|X_{fix}}^A$, and the resulting contingency tables are incorporated in equation (3). The exercises on random and perfect matching only use information from sample A . Restricting the sample to cover the same household types as in another year or region (sample B) is therefore not necessary; Assumption 1 is not required. However, in order to ensure that the conditional income distributions for the newly matched couples exist and the created overall income distribution function is well-defined, the following assumption needs to be satisfied in all exercises which use sorting algorithms.

Assumption 2 For all $x = (x_{E^m}, x_{E^f}, x_{fix}) \in E^m \times E^f \times X_{fix}$: $\mathbb{P}_{E^m, E^f|X_{fix}}^A(x) > 0$ if and only if $\mathbb{P}_{E^m, E^f|X_{fix}}(x) > 0$.

For the exercises on random and perfect sorting, the assumption is fulfilled when all conditional contingency tables $\mathbb{P}_{E|X_{fix}}^A$ of male and female education have positive entries. When the counterfactual mating pattern is taken from another year or region B , Assumption 2 is sufficient for generating a well-defined counterfactual income distribution. It states that a contingency table of male and female education has only a zero entry for A when its counterpart in B is zero as well, and vice versa.²⁴ However, the exercise becomes more comparable to the other exercises across time (space) when the sample is also restricted in order to fulfill Assumption 1. Therefore, I conduct the exercise in which the counterfactual mating pattern is taken from another year twice: once, to compare it to the exercises of random and perfect matching, and again, to compare it to the other exercises across time by omitting the household types which are not observed in either sample A or sample B .

²⁴In the East German subsample, there is no couple household in the age group 55 to 59 with a low-educated male (school) and a highly educated female (university). I overcome this problem by dropping all households for that age group in the exercise of interest.

4.3. Inequality Evaluation

The non-parametric framework produces a counterfactual income distribution for each exercise conducted. Because the whole counterfactual income distribution is known, any inequality measure can be used to evaluate the changes in the income distribution. Here, I utilize the Gini coefficient, the Theil index, and the Atkinson index.²⁵ The three different inequality measures exhibit different properties which enables a comprehensive analysis.²⁶ The Gini coefficient (Gini, 1921) is defined as

$$\mathfrak{G} := \frac{1}{2} - \int_0^1 \tilde{l}(y) dy,$$

where $\tilde{l} : [0, 1] \rightarrow [0, 1]$ denotes the piece-wise linear approximation of the Lorenz curve. It has the advantage that it is independent of income scale and population size. Its size is rather easy to interpret as it always ranges between 0 and 1. For the interpretation, note that the Gini coefficient puts most weight on the median.

In contrast, the Theil index (Theil, 1967) is more sensitive to the top of the distribution. It is also income scale-invariant. The upper bound of the Theil index depends on the sample; it amounts 100 times the logarithm of the number of distinct income levels. The Theil index considers the proportional relation between the income levels in the population. It is defined as

$$\mathfrak{T} := \int \frac{y}{\bar{y}} \log \left(\frac{y}{\bar{y}} \right) dF_Y(y),$$

where $\bar{y} = \int y dF_Y$.

The Atkinson's index (Atkinson, 1970) is based on the difference on marginal social utilities. Defined as

$$\mathfrak{A}_\varepsilon := 1 - \left[\int \left(\frac{y}{\bar{y}} \right)^{1-\varepsilon} dF_Y(y) \right]^{\frac{1}{1-\varepsilon}},$$

the Atkinson index features income scale independence and ranges between 0 and 1. If the inequality aversion parameter ε is large, the Atkinson index is sensitive to changes at the bottom of the income distribution.

²⁵Other inequality measures, which could also be applied, are percentile-ratios, coefficient of variation, Dalton's index, or other generalized entropy indices.

²⁶For a detailed description of the properties, please see Cowell (2011).

For improving the legibility, both the Gini coefficient, the Theil index as well as the Atkinson index are multiplied by the factor 100. This up-scaling lifts the upper bound of the Gini coefficient and the Atkinson index to 100.

By bootstrapping, I compute p -values for testing whether income inequality under the counterfactual scenario distinguishes from the inequality observed. I use bootstrapped p -values instead of standard errors, because the confidence intervals of the inequality measures are in general non-symmetric. Mills & Zandvakili (1997) show that statistical inference by bootstrapping performs well for the Gini coefficient and Theil's entropy measures. I follow their approach of the 'percentile method' which directly exploits the bootstrap distribution for the confidence intervals. For an extensive derivation of bootstrap methods, please see Efron & Tibshirani (1994).

I also evaluate the economic significance of the change in the Gini coefficient. I apply an approach presented in Blackburn (1989). He argues that an increase in the Gini coefficient can be linked to a lump sum tax k taken from every household below median income and given to all households above median income. The lump sum tax is given by

$$k = 2\bar{y} (\mathfrak{G}^{\text{data}} - \mathfrak{G}^{\text{counterfactual}}),$$

where $\bar{y} = \int y dF_Y^{\text{data}}$. The transfers are given in adult-equivalent income. For example, a decrease of the Gini coefficient from 0.30 to 0.23 accrues when every household above the median gives a lump sum of 14 percent of the mean income to all households below the median. Hence, in 2011, West Germany ($\mathfrak{G}^{2011} = 0.30$) would reach an inequality level from 1976 ($\mathfrak{G}^{1976} = 0.23$), if every household above the median income (1600 €) gives about 250 € to the households below the median income.

5. Results

5.1. West Germany across Time: 1976 versus 2011

Since 1976 income inequality has risen strongly in West Germany. After 35 years, the Gini coefficient is larger by around 7 points, the Atkinson index increased by more than 12 points and the Theil index almost doubled from 8.4 to 16.7 (see Table 4).²⁷ To a vast extent, changes in the socio-economic background of the society account for this increase

²⁷Here, and in all exercises which compare the impact of socio-economic characteristics from 1976 and 2011, household types are omitted if they are not observed in West Germany for either 1976 and 2011.

in inequality. Income inequality would be lower by around 4 Gini points if the socio-economic background of the households would have remained unchanged since the 1970s, but the households' income conditional on its socio-economic background is as in 2011. As the Lorenz curves in Figure 3 present, in particular, the share of poor households would diminish.

Table 4: Mean Income and Income Inequality

Region & year	Mean income	Income inequality		
		Gini coefficient	Theil index	Atkinson index ($\varepsilon = 2.0$)
West 1976 ¹	1305.41	22.98 (22.87, 23.11)	8.43 (8.33, 8.53)	16.49 (16.20, 16.78)
West 2011 ¹	1808.45	29.92 (29.67, 30.19)	16.69 (16.21, 17.12)	29.01 (28.50, 29.52)
East 2011 ²	1384.68	30.03 (29.52, 30.49)	15.59 (14.88, 16.32)	27.38 (26.57, 28.22)
West 2011 ²	1824.96	29.99 (29.74, 30.26)	16.77 (16.33, 17.21)	29.25 (28.73, 29.76)

Notes: 95% confidence intervals in parentheses. Mean income: in EURO, inflation-adjusted to 2011 levels and adjusted by the OECD-modified adult-equivalent scale.

¹ Household types are excluded if they are not observed in West Germany either in 1976 or 2011.

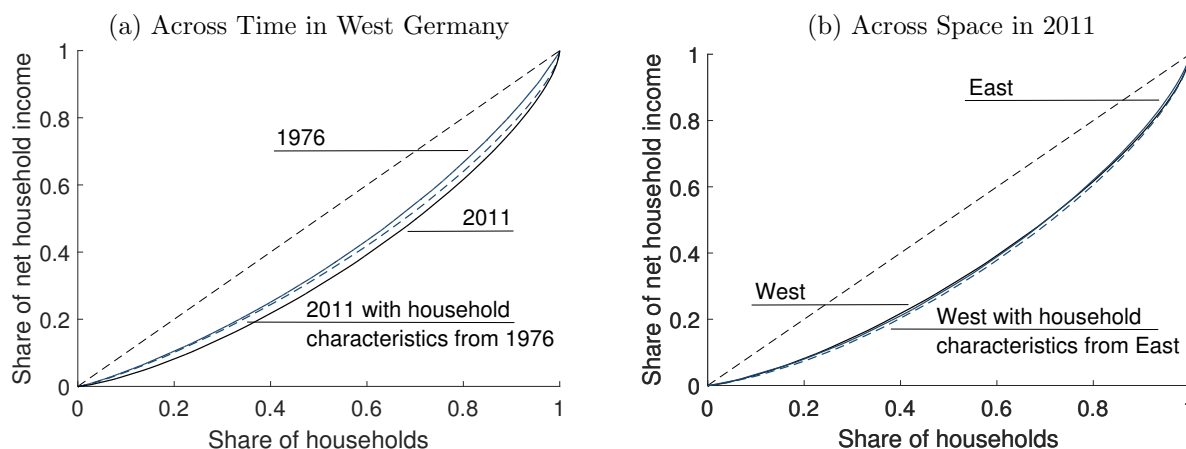
² Household types are excluded if they are not observed in 2011 for either East or West Germany.

Table 5 summarizes to what extent a change in a specific household characteristic, with all its indirect effects, contribute to that decline in income inequality.²⁸ Take, for example, the *age* distribution of the workforce. If in 2011 the society would be as young as in 1976, income inequality would barely drop; the Gini coefficient would decrease by only 0.23.

Contrary to the household's age distribution, the diminishing *household size* accounts for the rise in income inequality to a vast extent. If West German households were as rarely male single-headed or female single-headed as in 1976, income inequality would be lower by 1.76 Gini points. This decline in inequality is statistically and economically significant. Following the argument by Blackburn (1989), transfers of 63.66 € from households above

²⁸The entries in the table do not add up to the overall change in the Gini coefficient, i.e. 4.19, because of indirect effects; e.g. increasing the fraction of young households leads also to an increase in the number of children and may, therefore, affect the working status of married women which itself influences income inequality.

Figure 3: Distribution of Net Household Income



Notes: After omitting all household types which do not exist in either the factual or the counterfactual year and region. Income: adjusted by the OECD-modified adult-equivalent scale.

the median income to households below the median income would generate the same decline in inequality. The particularly strong change in the Atkinson index by 4.99 points out that this decline is pushed by a reduction of poor households. If, additionally, the conditional number of children in the households is as large as in 1976, the effect would be slightly amplified by further reducing the number of poor household.²⁹ Hence, the increase in single households drives the rise in inequality which is associated with declining households.³⁰ Although, in 2011, almost half of all West German households are run by singles, educational homogeneity, within the other half of the households, may have the potential to influence income inequality. First, in order to test whether positive assortative mating boosts inequality, suppose cohabiting males and females *sort into marriages* regarding their education in the same way as they did in the 1970s.³¹ Then, income inequality across West

²⁹This additional decline in inequality is only statistically significant for the Gini coefficient and the Atkinson index, and only at the five percent level.

³⁰All the findings on the household size, here and in the comparison across space, are independent of the adult-equivalent scale used; the square root scale and the old OECD equivalence scale (also called Oxford scale) provide the same results.

³¹The influence of marital sorting might be different when all single and non-single adults are re-matched. In practice, there is minor sorting into marriage by education. For example, in 2011, low-educated males and highly educated as well as low-educated females are more likely to be single (compare Table 2 and Table B.1 in the Appendix). Including these people from the tails of the educational distribution in the exercises makes education among married man (married females) more unequally distributed. Then, under random matching, the share of educational homogamous couples declines. As a consequence, the relative impact of marital sorting on income inequality would be overestimated.

Table 5: Income Inequality Change when West Germans in 2011 Have the Household Characteristics from 1976

Counterfactual characteristics from 1976	Gini coefficient	Theil index	Atkinson index ($\varepsilon = 2$)
All household characteristics	-4.19***	-4.14***	-9.11***
Age	-0.23	-0.26	-0.24
Household size			
Children	-0.09	-0.14	-0.31
Singles by gender	-1.76***	-2.07***	-4.99***
+ Children cond. <i>single</i> , gender, education	-1.72***	-2.04***	-4.95***
+ Children cond. cohabitation, gender, education	-2.11***	-2.42***	-5.61***
Education and educational homogamy			
Marital sorting	0.09	0.13	0.06
Marginal male and female education	-1.26***	-1.40***	-1.90***
Male and female education + marital sorting	-1.25***	-1.37***	-1.89***
Working status			
Married females working status	0.34*	0.41	0.03
Single females working status	-0.68***	-0.63**	-1.04***
+ Married females working status	-0.31	-0.21	-0.94**
Males working status	-0.76***	-0.69**	-1.42***
+ Females working status	-1.03***	-0.86***	-2.35***
Working status + education incl. sorting	-2.47***	-2.25***	-4.46***
Overall difference in income inequality between 1976 and 2011	-6.94***	-8.26***	-12.51***

Notes: Difference: counterfactual minus factual inequality measure. Two-sided significance test: *** $p < 1\%$, ** $p < 5\%$, * $p < 10\%$.

German households would not differ from the observed inequality in 2011. Second, suppose the more extreme case in which males and females in 2011 are matched randomly in terms of education. Then, income inequality would be lower than observed in 2011, but this decline in inequality would still not be statistically nor economically significant (see Table 6, “data vs. random matching”). Only in the more extreme case in which the scenario of randomly matched couples is compared to the scenario of perfectly matched couples, the Gini coefficient changes statistically significantly (see Table 6). However, the increase in inequality associated with the change from random to perfect matching is not economically significant. The same effect on income inequality would occur if 1 percent of the mean income, that are 18.08 €, is taken from every household below the median income level and transferred to the households above the median income level.

Table 6: Differences in Inequality under Distinct Mating Schemes

	West 2011	East 2011
Data vs. random matching		
Gini coefficient	0.27	0.26
Theil index	0.34	0.43
Atkinson index with parameter 2.0	0.25	0.08
Perfect matching vs. data		
Gini coefficient	0.23	0.21
Theil index	0.32	0.39
Atkinson index with parameter 2.0	0.21	0.04
Perfect vs. random matching		
Gini coefficient	0.50***	0.47
Theil index	0.66**	0.83
Atkinson index with parameter 2.0	0.46	0.12

Notes: Before omitting any household type. Inequality measures: multiplied by 100. One-sided significance test with p -values: *** $p < 1\%$, ** $p < 5\%$, * $p < 10\%$.

Contrary to marital sorting in education, changes in the populations’ *education* significantly contribute to the rise in income inequality. Changes in male and female education account for approximately a fifth of the inequality increase between 1976 and 2011. In the 1970s, education is very pronounced at the lower levels. Because education and income are highly correlated, the unilateral distribution of education from the 1970s is associated with less variation in the population’s income distribution. The counterfactual exercise suggests that if West Germans in 2011 have the same education as their ancestors in 1976, income

inequality would diminish by 1.26 Gini points.

The *employment* situation of 1976 also produces less income inequality than the employment situation in 2011. If the West German households of 2011 face the same employment as the households of 1976, then income inequality would shrink by 1.03 Gini points. In particular, the reduced employment of males widens the income gap; income inequality would already shrink by 0.76 Gini points if only the males face the same employment as in 1976. Counterfactual analysis on female's employment suggest that, first, raising the female labor supply among singles to the levels of 1976 reduces income inequality by lifting the household income of single females.³² And, second, the employment of married females equalizes income more in 2011 than in the 1970s. The increased (part-time) employment of married females slightly offsets the increased inequality arising by their husbands' incomes. However, the stable Atkinson index shows that the rise in the employment of married females has not reduced the number of poor households, but only rearranged income in the middle income households.

5.2. Germany in 2011 across Space: East versus West

At first sight, in 2011, income inequality in East and West Germany are very similar. For both regions, the Gini coefficient lies around 30 points (see Table 4). However, there are some deviations at the tails. Both a lower Atkinson index and a lower Theil index indicate less income inequality in the East. Socio-economic factors may account for such mismatches in the income distribution. Can the variations in the socio-economic background of East and West German households also account for the lower Atkinson index and the lower Theil index in the East? The answer is no. Both indices increase for West Germany if West German households would face the same socio-economic background as East Germans. The Atkinson index would increase strongly by 3.43 points, and even the Gini coefficient would increase by statistically and economically significant 1.88 points (see Table 7 and, for a visualization, see Figure 3).³³ Hence, the socio-economic background of the East German households produces more income inequality than the socio-economic background of the West German households. This discrepancy is more than offset by the East German conditional income distribution.

³²See Table B.2 in the Appendix, for the share of full-time working, part-time working and jobless single female households by income quantiles.

³³Here, and in all exercises which compare the impact of socio-economic characteristics from East and West Germany, household types are omitted if they are not observed in 2011 for both sample regions East and West.

Knowing that the East German household characteristics makes income more unequally distributed whereas the West German household characteristics equalize income, the following question comes into mind: How does each socio-economic characteristic account for the different consequences on income inequality? Several counterfactual exercises on West German income inequality are conducted to answer this question. (Analogue exercises on East German income inequality are also conducted. The results are listed in Table B.5 in the Appendix. They show similar effects but of reversed sign and smaller magnitude. However, because of the smaller sample size for East Germany, the results are less statistically significant.)

The distinct *age* distributions across East and West German households do not account for different levels of income inequality. For income inequality in the West, it does not matter whether the working age population is as young as West Germans or as old as East Germans. Table 7 summarizes the shifts in the Gini coefficient, the Theil index, and the Atkinson index which result from this and all the following exercises. For example, in Table 7 the entry of the line “Age” shows that if West German households are as old as East Germans, income inequality would decline by insignificant 0.19 Gini points.

A minor in disequalizing effect has the relatively small *household size* in East Germany. If both the share of male and female single-headed households and the number of children living in the households are the same in West as in East Germany, then income inequality in West Germany would increase by 0.65 Gini points. The impact of the children dominates this change in income inequality. More children in single households may lead to an increase in poor households, and fewer children in couple households may lead to an increase in high-income households. Both changes boost income inequality.

Similar to West Germany (see Section 5.1), *marital sorting* in formal education has no potential to influence income inequality in East Germany (see Table 6). Even in the extreme scenario in which income inequality in the case of random matching is compared to the case of perfect matching, the surplus of inequality is not significantly different.

The small variation in East German *education* leads to a lower level of income inequality across households. If West Germans are, to the favor of a larger share of vocationally trained people, as rarely low-educated and as rarely highly educated as their East German peers, the Gini coefficient would diminish by 1.38 points. Following the argument by Blackburn (1989), this reduction in inequality is the same as when taking 50.37 € from each household above the median income level and giving it to the households below the

Table 7: Income Inequality Change when West Germans in 2011 Have the Household Characteristics from the East

Counterfactual characteristics from the East	Gini coefficient	Theil index	Atkinson index ($\varepsilon = 2$)
All household characteristics	1.88***	1.77***	3.43***
Age	0.19	0.19	0.28
Household size			
Children	0.54***	0.54*	0.58
Singles by gender	0.25	0.32	0.70*
+ Children cond. <i>single</i> , gender, education	0.43**	0.45	0.76**
+ Children cond. cohabitation, gender, education	0.65***	0.72**	1.06**
Education and educational homogamy			
Marital sorting	0.02	0.03	0.03
Marginal male and female education	-1.38***	-1.43***	-2.08***
Male and female education + marital sorting	-1.38***	-1.42***	-2.08***
Working status			
Married females working status	0.44**	0.48	0.75**
Single females working status	0.65***	0.60*	1.08***
+ Married females working status	1.10***	1.09***	1.85***
Males working status	1.18***	1.11***	2.36***
+ Females working status	2.32***	2.29***	4.11***
Working status + education incl. sorting	0.93***	0.77**	1.99***
Overall difference in income inequality between East and West	0.04	-1.18**	-1.87***

Notes: Difference: counterfactual minus factual inequality measure. Two-sided significance test: *** $p < 1\%$, ** $p < 5\%$, * $p < 10\%$.

median income level. This equalizing impact of education is opposed to a heavily disequalizing employment situation in East Germany.³⁴

In fact, among all household characteristics studied, the *working status* has the strongest impact on income inequality. If West Germans face the same working status as their East German peers, then income inequality would increase by substantially 2.32 Gini points.³⁵ The Atkinson index would even rise by 4.11 points indicating a strong increase in low income households. This rise in low income households can be explained by the relatively large share of jobless males and single females in East Germany, who are associated with low income. Because the difference in the share of jobless people is larger between East and West German males than between East and West German females, the Atkinson index is more influenced by male than by female employment. Moreover, an exercise on the employment of married females indicates that inequality would rise if the West German wives increase their employment attachment to the level of East Germans. This result stands in contrast to the findings across time (see Section 5.1). Whereas the increased employment attachment of West German females over time is interpreted as a stronger attempt to offset the inequality arising by the husband's income, East German females seem to work more than their West German peers because of another reason than equalizing income variations across households. However, the driving forces behind female employment decisions are beyond the scope of this paper.

6. Discussion and Conclusion

The aim of this paper is a better understanding of the connection between the socio-economic characteristics of households and net income inequality across the households. I apply German microcensus data from 1976 and 2011 for a reweighting analysis which includes transformations in marital sorting. First, I quantify by how much changes in the households' socio-economic characteristics, like age, household size, education and employment, contribute to the West German income inequality rise since the 1970s. Then, I investigate by how much East-West-German differences in these household characteristics influence income inequality in the two German regions in 2011.

³⁴For the joint impact of the East German employment, education, and marital sorting, see line "Working Status + education incl. sorting" in Table 7. As outlined before, the impact of marital sorting is negligible.

³⁵Also, the other way around, if the conditional employment of East Germans coincides with that of their West German peers, income inequality would be affected strongly (see Table B.5 in the Appendix).

In the analysis over time, I find that the prevalence of singlehood plays a major role for the rise in income inequality. Besides, changes in education and employment contribute to the rise in income inequality, although by a smaller magnitude. I could not find a significant effect of the older society suggesting that the age distribution of the workforce changed too little since the 1970s to influence income inequality significantly.

The analysis on East and West Germany shows that overall income inequality in 2011 is similar in both regions. However, the East German employment pattern promotes income inequality stronger than the West German employment pattern. The main reason is the presence of many jobless males and jobless single females. Creating full-time jobs may reduce the income inequality in the East. Besides employment, the smaller household size in East Germany slightly increases income inequality. Contrary to the findings on the temporal comparison, here, the inequality effect of the declining household size is driven by the difference in the number of children. Fewer children in couple households and more children in single households widen the income gap. Special support for single parents may be able to diminish such a gap. In East Germany, two factors work against the amplification of income inequality by employment and household size. First, education within East German households is rather similar. I argue that this low variation in education creates less variation in income by influencing labor income. Second, conditional on the household characteristics, East German income is less dispersed than in the West.

The exercises illustrate the importance of differentiating between married and single female employment when considering the impact on household income. In particular, the employment of married females affects income inequality in two ways: either their income offsets the income inequality arising by their husbands' employment or it amplifies the income inequality across households. The West German rise in married female employment, if at all, diminishes income inequality across households whereas inequality would grow if the married females in West German increase their employment attachment to East German levels. The ambiguous effects of rising labor force participation among married females allows for no policy implications; they emphasize the need for further research on the employment incentives of married females.

Neither across East nor across West German households, I could find any evidence that marital sorting in formal education boosts net income inequality. For West Germany, Pestel (2016) provides an explanation for this finding. He argues that the low labor force participation of married females makes sorting in earnings potentials irrelevant for earnings inequality. However, for East Germany, he finds that sorting in earnings potentials strongly

affects earnings inequality. My results suggest that either the German tax and transfer system works against that effect on earnings inequality (Fuchs-Schündeln et al., 2010) or East German couples sort not only in education, but also in other earnings potentials like, for example, occupation or industry. However, both an analysis of the effects of the German tax and transfer system and a decomposition of marital sorting in several earnings potentials is beyond the scope of this paper and left for future research.

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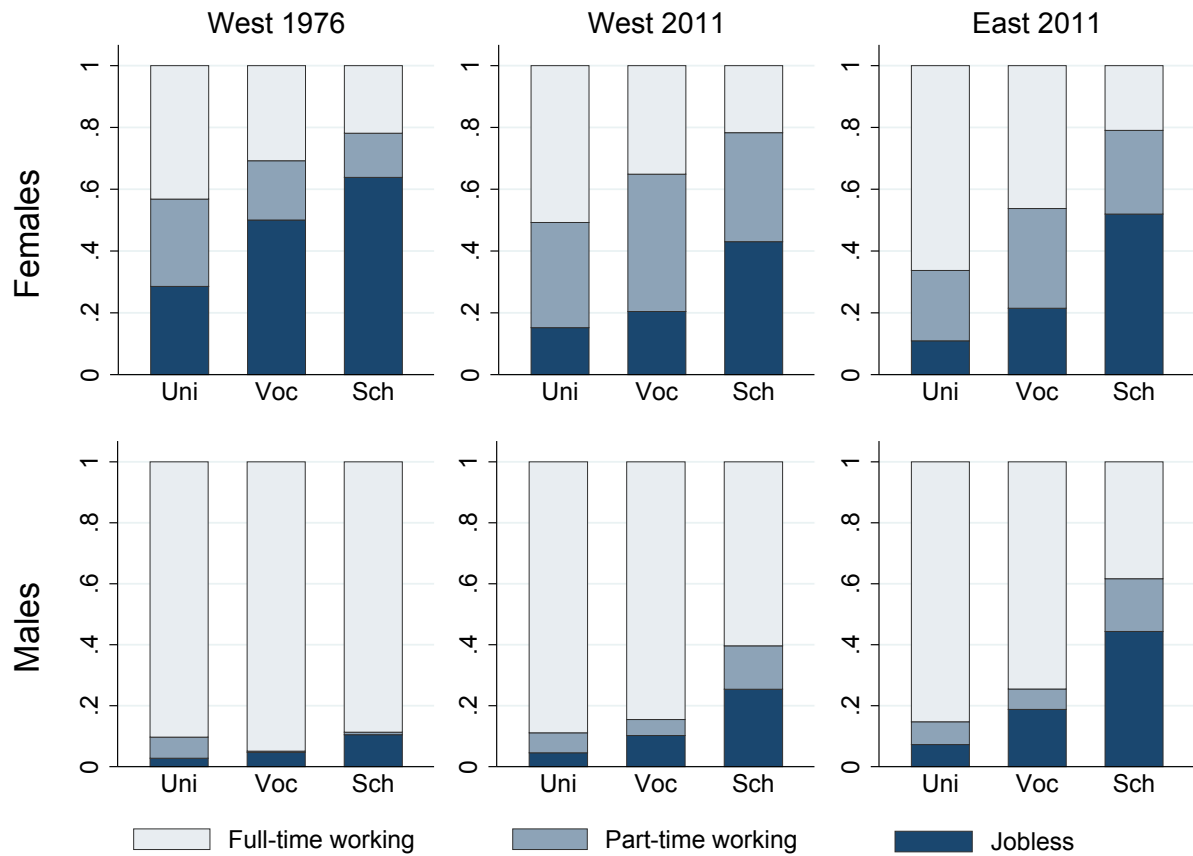
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Appendix

A. Figures

Figure A.1: Employment by Education and Gender



B. Tables

Table B.1: Contingency Tables Of Couples' Education

West Germany 1976		Females			<i>Males total</i>
		University	Apprenticeship	School	
Males	University	2.42	4.88	1.60	<i>8.90</i>
	Apprenticeship	0.52	41.57	29.72	<i>71.81</i>
	School	0.14	3.64	15.52	<i>19.30</i>
<i>Females total</i>		<i>3.08</i>	<i>50.09</i>	<i>46.84</i>	

West Germany 2011		Females			<i>Males total</i>
		University	Apprenticeship	School	
Males	University	12.86	11.23	1.19	<i>25.28</i>
	Apprenticeship	5.13	52.73	8.03	<i>65.90</i>
	School	0.73	3.92	4.17	<i>8.82</i>
<i>Females total</i>		<i>18.73</i>	<i>67.88</i>	<i>13.39</i>	

East Germany 2011		Females			<i>Males total</i>
		University	Apprenticeship	School	
Males	University	9.88	8.47	0.41	<i>18.76</i>
	Apprenticeship	6.48	69.92	2.36	<i>78.76</i>
	School	0.37	1.16	0.95	<i>2.48</i>
<i>Females total</i>		<i>16.73</i>	<i>79.55</i>	<i>3.72</i>	

Notes: Fraction of all married households, in percent.

Table B.2: Share of Single Female Households in the Top / Bottom of the 2011 West / East German Income Distribution, by Employment

Part of the income distribution	West German single females			East German single females		
	Full-time work	Part-time work	Jobless	Full-time work	Part-time work	Jobless
Top 5 percent	8.22	1.17	0.00	8.51	0.00	0.00
Top 10 percent	6.02	0.63	0.00	8.31	0.20	0.00
Top 25 percent	10.58	0.62	0.07	6.61	0.40	0.00
Top 50 percent	13.29	0.97	0.15	9.78	0.67	0.08
Bottom 50 percent	14.22	7.87	5.67	10.75	7.50	8.66
Bottom 25 percent	12.69	11.10	8.83	8.56	10.09	13.11
Bottom 10 percent	7.32	14.88	15.63	3.30	8.99	18.74
Bottom 5 percent	4.41	14.15	20.91	2.15	5.33	10.93
Total	13.95	5.82	4.03	10.46	5.46	6.10

Notes: Fraction, in percent.

Table B.3: Coverage of Household Types

Households in the West in 2011 with no counterpart in 1976:	2.06 %
Households in the West in 2011 with no counterpart in the East (2011):	3.96 %
Households in the East in 2011 with no counterpart in the West (2011):	0.14 %

Table B.4: Income Inequality before and after Approximations.

Income inequality	West 2011	East 2011
Gini coefficient		
Before approximation: factual data	30.11	30.06
1st approximation: maximum number of children	30.09	30.04
2nd approximation (across time): dropping households	29.92	-
2nd approximation (across space): dropping households	29.99	30.03
Theil index		
Before approximation: factual data	16.88	15.62
1st approximation: maximum number of children	16.86	15.60
2nd approximation (across time): dropping households	16.69	-
2nd approximation (across space): dropping households	16.77	15.59
Atkinson index with parameter 2.0		
Before approximation: factual data	29.17	27.41
1st approximation: maximum number of children	29.14	27.39
2nd approximation (across time): dropping households	29.01	-
2nd approximation (across space): dropping households	29.25	27.38

Notes: In the first approximation, households with three or more children are made count as households with two children. In the second approximation, household types are omitted if they are not observed in the counterfactual year and region.

Table B.5: Income Inequality Change when East Germans in 2011 Have the Household Characteristics from the West

Counterfactual characteristics from the West	Gini coefficient	Theil index	Atkinson index ($\varepsilon = 2$)
All	-1.15***	-0.89*	-1.97***
Age	-0.17	-0.15	-0.17
Household size			
Children	-0.34	-0.38	-0.42
Singles by gender	-0.24	-0.27	-0.46
+ Children cond. <i>single</i> , gender, education	-0.31	-0.33	-0.39
+ Children cond. cohabitation, gender, education	-0.68**	-0.72	-0.96
Education and educational homogamy			
Marital sorting	0.02	0.01	0.02
Marginal male and female education	1.32***	1.37***	1.90**
Male and female education + marital sorting	1.41***	1.46***	1.97***
Working status			
Married females working status	-0.29	-0.17	-0.58
Single females working status	-0.46	-0.42	-0.58
+ Married females working status	-0.74**	-0.58	-1.14**
Males working status	-1.21***	-1.08**	-1.75***
+ Females working status	-1.93***	-1.62***	-2.94***
Working status + education incl. sorting	-0.59*	-0.32	-1.01*
Overall difference in income inequality between West and East	-0.04	1.18**	1.87***

Notes: Difference: counterfactual minus factual inequality measure. Two-sided significance test: *** $p < 1\%$, ** $p < 5\%$, * $p < 10\%$.

C. Detailed Description of the Counterfactual Exercises

Exchanging All Household Characteristics

How strong would income inequality change if households in 2011 have the same characteristics as in the 1970s, but their household income conditional on the characteristics are distributed as in 2011? That is, households' age, their probability of being single, their number of children, education, and employment are as in the 1970s. With A standing for West Germany in 2011 and B being West Germany in 1976, the following income distribution takes into account both the conditional income of A and the households characteristics of B :

$$F_Y(I) = \int_0^I \sum_x f_{Y|X}^A(y, x) \mathbb{P}_X^B(x) dy.$$

Note that F_Y is only well-defined under the following assumption.

Assumption C.1 For all $x \in X$: $\mathbb{P}_X^A(x) = 0$ if and only if $\mathbb{P}_X^B(x) = 0$.

The assumption is fulfilled when all household types which cover at least one household from region and year A are also represented by at least one household from region and year B , and vice versa.³⁶

Exercises on the Households Age

If households in 2011 are as young as in the 1970s, would income inequality decline? Increasing the fraction of young households, without taking into account that young households differ in their number of kids, employment status, and cohabitation from old households might provide misleading results. Holding all other household characteristics constant conditional on the households' age while adjusting the age of the households, provides a more intuitive counterfactual income distribution. With A standing for West Germany in 2011 and B being West Germany in 1976, the counterfactual income distribution is given by

$$F_Y(I) = \int_0^I \sum_{x_{-H} \in X_{-H}} \sum_{x_H \in H} f_{Y|X}^A(y, (x_{-H}, x_H)) \mathbb{P}_{X_{-H}|H}^H(x_{-H}, x_H) \mathbb{P}_H^B(x_H) dy,$$

³⁶As it is summarized in Table B.4, deleting all households in the subsamples (West Germany 1976 and 2011, and East Germany 2011) which do not fulfill this assumption decreases the number of observations by less than 4%. Moreover, it does not change income inequality essentially.

where $x_{-H} \in X_{-H}$ and $X_{-H} = W \times K \times E \times G \times C$, that is the set of household types when it is not controlled for age. $\mathbb{P}_{X_{-H}|H}^i(x_{-H}, x_H)$ denotes the conditional probability that a household of year or region i ($i = A, B$) has the household type $x = (x_{-H}, x_H)$ given that that household is of age group $x_H \in H$. The probability that the household belongs to age group x_H is given by $\mathbb{P}_H^i(x_H)$.

The following assumption ensures that F_Y is well-defined.

Assumption C.2 For all $x_H \in H$: $\mathbb{P}_H^A(x_H) = 0$ if and only if $\mathbb{P}_H^B(x_H) = 0$.

For the German microcensus used, the assumption is fulfilled because for every age group there exist a household belonging to that age group.

Exercises on Singlehood, Single Parents and Households Size

How would the income distribution change if singlehood in West Germany in 2011 is as low (high) as in 1976 (East Germany)? As the cohabitation status should not modify the age distribution across households, the households age needs to be held constant. On the contrary, singles have fewer children than couples and couples, in particular married females, make different labor supply decisions such that their working status differs from single females. Ignoring these effects in a counterfactual exercise in which the share of single households is reduced may lead to strongly deviating results. To allow for the different indirect effects of singles by gender, not only the overall share of singles but the share of single females and single males is adjusted. A counterfactual income distribution function which considers all the above criteria follows with

$$F_Y(I) = \int_0^I \sum_{x \in X} f_{Y|X}^A(y, (x_W, x_K, x_E, x_G, x_C, x_H)) \mathbb{P}_{W,K,E|G,C,H}^A(x_W, x_K, x_E, x_G, x_C, x_H) \mathbb{P}_{G,C|H}^B(x_G, x_C, x_H) \mathbb{P}_H^A(x_H) dy,$$

where $\mathbb{P}_{W,K,E|G,C,H}^A$ and $\mathbb{P}_{G,C|H}^B$ represent conditional shares of the population in A and B , respectively. Among all households of age x_H , the share of household with cohabitation status x_C and gender x_G is given by $\mathbb{P}_{G,C|H}^i(x_G, x_C, x_H)$ (for $i = A, B$). Here and hereafter, the interpretation of other conditional shares $\mathbb{P}_{\cdot|\cdot}^i$ follows analogously. For F_Y^i to be well-defined, a weak version of Assumption C.1 is required.

Assumption C.3 For all $x = (x_G, x_C, x_H) \in G \times C \times H$ with $\mathbb{P}_H^A(x_H) > 0$: $\mathbb{P}_{G,C|H}^A(x) = 0$ if and only if $\mathbb{P}_{G,C|H}^B(x) = 0$.

By applying the rich Microcensus data Assumption C.3 is satisfied as it states the following: Suppose in the categorization of household types, it is only controlled for age, cohabitation, and gender. If there is any combination of gender, cohabitation status, and age group which cannot be found among the households for region and year A , then that combination should also not be represented for region and year B , and vice versa.

Among all the singles, the likelihood of having children also differs between East and West Germany, as well as across time. Taking this into account might give a better answer to the question how single parenting affects income inequality. The according counterfactual income distribution is given by

$$F_Y(I) = \int_0^I \sum_{x \in X} f_{Y|X}^A(y, (x_W, x_K, x_E, x_G, x_C, x_H)) \mathbb{P}_{W|K,E,G,C,H}^A(x_W, x_K, x_E, x_G, x_C, x_H) \mathbb{P}_{K|E,G,C,H}^*(x_K, x_E, x_G, x_C, x_H) \mathbb{P}_{E|G,C,H}^A(x_E, x_G, x_C, x_H) \mathbb{P}_{G,C|H}^B(x_G, x_C, x_H) \mathbb{P}_H^A(x_H) dy,$$

where

$$\mathbb{P}_{K|E,G,C,H}^*(x_K, x_E, x_G, x_C, x_H) = \begin{cases} \mathbb{P}_{K|E,G,C,H}^B(x_K, x_E, x_G, x_C, x_H) & \text{if } x_C = \text{single} \\ \mathbb{P}_{K|E,G,C,H}^A(x_K, x_E, x_G, x_C, x_H) & \text{if } x_C = \text{couple} . \end{cases}$$

Thereby, singlehood is still conditioned on the household heads' age and the number of children is additionally conditioned on the households gender and education category. Assumption C.1 guarantees a well-defined counterfactual income distribution for all exercises in which the conditional number of children is adjusted. With

$$\mathbb{P}_{K|E,G,C,H}^*(x_K, x_E, x_G, x_C, x_H) = \mathbb{P}_{K|E,G,C,H}^B(x_K, x_E, x_G, x_C, x_H),$$

a counterfactual income distribution is generated for quantifying the impact of household size. Thereby, both the share of single households by gender and the conditional number

of children living in single and married households is adjusted. Considering only the effect of changes in the conditional number of children living in the households, $\mathbb{P}_{K|E,G,C,H}^*$ is plugged into

$$F_Y(I) = \int_0^I \sum_{x \in X} f_{Y|X}^A(y, (x_W, x_K, x_E, x_G, x_C, x_H)) \mathbb{P}_{W|K,E,G,C,H}^A(x_W, x_K, x_E, x_G, x_C, x_H) \mathbb{P}_{K|E,G,C,H}^*(x_K, x_E, x_G, x_C, x_H) \mathbb{P}_{E,G,C,H}^A(x_E, x_G, x_C, x_H) dy.$$

Exercises on Education and Marital Sorting

For both marital sorting and gender-specific education, I construct counterfactual exercises in which the populations' age distribution, the share of single households, and their gender are held constant. For year or region A , the counterfactual income distribution is given by

$$F_Y(I) = \int_0^I \sum_{x \in X} f_{Y|X}^A(y, (x_W, x_K, x_E, x_G, x_C, x_H)) \mathbb{P}_{W,K|E,G,C,H}^A(x_W, x_K, x_E, x_G, x_C, x_H) \mathbb{P}_{E|G,C,H}^*(x_E, x_G, x_C, x_H) \mathbb{P}_{G,C,H}^A(x_G, x_C, x_H) dy, \quad (\text{C.1})$$

where $\mathbb{P}_{E|G,C,H}^*(x_E, x_G, x_C, x_H)$ is the counterfactual share of households with educational level x_E among all households of age x_H , cohabitation x_C , and gender x_G . Compared to the factual data, conditional education of couples are replaced in such a way that either marital sorting or marginal distribution of male and female education are fixed.

I conduct several counterfactual exercises on marital sorting. In the first exercise, I test whether random matching would diminish income inequality. In the second exercise, I assume an educational sorting pattern from another year or region B and test whether marital sorting differs so much that it has a significant impact on income inequality. Finally, I test whether perfect positive assortative matching, in the sense of the Gale-Shapley matching algorithm (Gale & Shapley, 1962), would increase income inequality significantly. For singles, conditional education in Equation (C.1) does not differ from the factual data, but the sorting algorithms affect the conditional joint education $x_E = (x_E^m, x_E^f) \in E^f \times E^m$

of couples:

$$\mathbb{P}_{E|G,C,H}^*(x_E, x_G, x_C, x_H) = \begin{cases} \mathbb{P}_{E|G,C,H}^{A, \text{sort}}(x_E, x_G, x_C, x_H) & \text{if } x_C = \text{couple} \\ \mathbb{P}_{E|G,C,H}^A(x_E, x_G, x_C, x_H) & \text{if } x_C = \text{single} . \end{cases} \quad (\text{C.2})$$

$\mathbb{P}_{E|G,C,H}^{A, \text{sort}}(\cdot, \cdot, \text{couple}, x_H)$ represents a contingency table for each age group x_H . Matching algorithms are used to create counterfactual contingency tables $\mathbb{P}_{E|G,C,H}^{A, \text{sort}}$. Thereby, male and female conditional marginal education, $\mathbb{P}_{E^m|G,C,H}^A$ and $\mathbb{P}_{E^f|G,C,H}^A$, respectively, remain unchanged. For the exercise on random sorting, the contingency tables are replaced by

$$\mathbb{P}_{(E^m, E^f)|G,C,H}^{A, \text{sort}}(x_{E^m}, x_{E^f}, x_G, x_C, x_H) = \mathbb{P}_{E^m|G,C,H}^A(x_{E^m}, x_G, x_C, x_H) \mathbb{P}_{E^f|G,C,H}^A(x_{E^f}, x_G, x_C, x_H).$$

In order to adopt the educational sorting pattern from a different year or region B , the contingency tables $\mathbb{P}_{E|G,C,H}^B$ are transformed with the Sinkhorn-Knopp algorithm. The sorting pattern of B is fixed, but male and female marginal conditional education from A is incorporated. The transformed contingency tables are inserted as $\mathbb{P}_{E|G,C,H}^{A, \text{sort}}$ in equation (C.2). For the exercise on perfect sorting, the Gale-Shapley algorithm is applied on $\mathbb{P}_{E|G,C,H}^A$, and the resulting contingency tables are inserted as $\mathbb{P}_{E|G,C,H}^{A, \text{sort}}$ in equation (C.2). The exercises on random and perfect matching only use information from sample A . Restricting the sample to cover the same household types as in another year or region (sample B) is therefore not necessary; Assumption 1 is not required. However, in order to ensure that the conditional income distributions for the newly matched couples exist and the created overall income distribution function is well-defined, the following assumption needs to be satisfied in all exercises which use sorting algorithms.

Assumption C.4 For all $x = (x_{E^m}, x_{E^f}, x_G, x_C, x_H) \in E^m \times E^f \times G \times C \times H$: $\mathbb{P}_{E^m, E^f|G,C,H}^A(x) > 0$ if and only if $\mathbb{P}_{E^m, E^f|G,C,H}(x) > 0$.

For the exercises on random and perfect sorting, the assumption is fulfilled when all conditional contingency tables $\mathbb{P}_{E|X_{fix}}^A$ of male and female education have positive entries. When the counterfactual mating pattern is taken from another year or region B , Assumption C.4 is sufficient for generating a well-defined counterfactual income distribution. It states that a contingency table of male and female education has only a zero entry for A when its counterpart in B is zero as well, and vice versa.³⁷ However, the exercise becomes more

³⁷In the East German subsample, there is no couple household in the age group 55 to 59 with a low-educated male (school) and a highly educated female (university). I overcome this problem by dropping all households for that age group in the exercise of interest.

comparable to the other exercises across time (space) when the sample is also restricted in order to fulfill Assumption 1. Therefore, I conduct the exercise in which the counterfactual mating pattern is taken from another year twice: once, to compare it to the exercises of random and perfect matching, and again, to compare it to the other exercises across time by omitting the household types which are not observed in either sample A or sample B . Consider the overall increase in education from 1976 to 2011. How does that change in education affect income inequality?

For the counterfactual exercise, the marginal distribution of male and female education conditional on age is taken from another region or year. Then, the counterfactual share of households with education x_E given their age x_H , gender x_G , and cohabitation status x_C is given by

$$\mathbb{P}_{E|G,C,H}^*(x_E, x_G, x_C, x_H) = \begin{cases} \mathbb{P}_{E|G,C,H}^{B, \text{SK}(A)}(x_E, x_G, x_C, x_H) & \text{if } x_C = \text{couple} \\ \mathbb{P}_{E|G,C,H}^B(x_E, x_G, x_C, x_H) & \text{if } x_C = \text{single} . \end{cases}$$

$\mathbb{P}_{E|G,C,H}^{B, \text{SK}(A)}(\cdot, \cdot, \text{couple}, x_H)$ denotes the contingency table regarding conditional education of husbands and wives after the Sinkhorn-Knopp (SK) algorithm has been applied to the mating scheme of A with the marginal distributions of male and female education from B . Without the SK algorithm, that is, if

$$\mathbb{P}_{E|G,C,H}^*(x_E, x_G, x_C, x_H) = \mathbb{P}_{E|G,C,H}^B(x_E, x_G, x_C, x_H),$$

both the educational sorting within married households and the conditional distribution of male and female education are supposed to be as in B . In both the exercise with the SK algorithm and the exercise without the SK algorithm, Assumption C.4 is sufficient for ensuring the construction of a well-defined counterfactual income distribution. It is satisfied when a contingency table of year and region A has only zero entries if its counterpart in B (after applying the SK algorithm) is zero as well, and vice versa.

Exercises on Employment

As pointed out in Section 3, employment influences how changes in households' age, singlehood, and education affect income inequality. In the 1970s, most married females were jobless. Until 2011, this share more than halved. East German married females are by 20 percent more often full-time working than West Germans. How much does the shift in

conditional employment affect income inequality? The influence of the different (female) employment situations in East and West Germany, and in the 1970s and 2011, is computed with the counterfactual income distribution given by

$$F_Y(I) = \int_0^I \sum_{x \in X} f_{Y|X}^A(y, (x_W, x_K, x_E, x_G, x_C, x_H)) \mathbb{P}_{W|K,E,G,C,H}^*(x_W, x_K, x_E, x_G, x_C, x_H) \mathbb{P}_{K,E,G,C,H}^A(x_K, x_E, x_G, x_C, x_H) dy.$$

With $x := (x_W, x_K, x_E, x_G, x_C, x_H)$, conditional employment of married females is adjusted by applying

$$\mathbb{P}_{W|K,E,G,C,H}^*(x) = \begin{cases} \mathbb{P}_{W^f|W^m,K,E,G,C,H}^B(x) \mathbb{P}_{W^m|K,E,G,C,H}^A(x) & \text{if } x_C = \text{couple} \\ \mathbb{P}_{W|K,E,G,C,H}^A(x) & \text{if } x_C = \text{single} \end{cases}.$$

Thereby, conditional employment of males and single females is held constant. Analogously, with

$$\mathbb{P}_{W|K,E,G,C,H}^*(x) = \begin{cases} \mathbb{P}_{W|K,E,G,C,H}^A(x) & \text{if } x_C = \text{couple} \\ \mathbb{P}_{W|K,E,G,C,H}^B(x) & \text{if } x_C = \text{single}, x_G = \text{female} \\ \mathbb{P}_{W|K,E,G,C,H}^A(x) & \text{if } x_C = \text{single}, x_G = \text{male} \end{cases}$$

conditional employment of single females is adopted while the conditional employment of males and married females is fixed. Together,

$$\mathbb{P}_{W|K,E,G,C,H}^*(x) = \begin{cases} \mathbb{P}_{W^f|W^m,K,E,G,C,H}^B(x) \mathbb{P}_{W^m|K,E,G,C,H}^A(x) & \text{if } x_C = \text{couple} \\ \mathbb{P}_{W|K,E,G,C,H}^B(x) & \text{if } x_C = \text{single}, x_G = \text{female} \\ \mathbb{P}_{W|K,E,G,C,H}^A(x) & \text{if } x_C = \text{single}, x_G = \text{male} \end{cases}$$

takes the conditional employment of all females from region and year B into account. Analogously, a counterfactual exercise on male employment is conducted with

$$\mathbb{P}_{W|K,E,G,C,H}^*(x) = \begin{cases} \mathbb{P}_{W^f|W^m,K,E,G,C,H}^A(x) \mathbb{P}_{W^m|K,E,G,C,H}^B(x) & \text{if } x_C = \text{couple} \\ \mathbb{P}_{W|K,E,G,C,H}^A(x) & \text{if } x_C = \text{single}, x_G = \text{female} \\ \mathbb{P}_{W|K,E,G,C,H}^B(x) & \text{if } x_C = \text{single}, x_G = \text{male} \end{cases}$$

Applying

$$\mathbb{P}_{W|K,E,G,C,H}^*(x) = \mathbb{P}_{W|K,E,G,C,H}^B(x)$$

provides a counterfactual distribution which supposes the conditional employment of all household members from the counterfactual year and region B .

Is the effect of education amplified by the change in employment or reduced? Both together, the change in education (including marital sorting) and the different conditional working status can reinforce income inequality or their impacts on income inequality can compensate each other. In order to find an answer to this question, the counterfactual income distribution

$$F_Y(I) = \int_0^I \sum_{x \in X} f_{Y|X}^A(y, (x_W, x_K, x_E, x_G, x_C, x_H)) \mathbb{P}_{W|K,E,G,C,H}^B(x_W, x_K, x_E, x_G, x_C, x_H) \mathbb{P}_{K|E,G,C,H}^A(x_W, x_K, x_E, x_G, x_C, x_H) \mathbb{P}_{E|G,C,H}^B(x_W, x_K, x_E, x_G, x_C, x_H) \mathbb{P}_{G,C,H}^A(x_K, x_E, x_G, x_C, x_H) dy$$

is generated. Note that exercises on employment require the Assumption C.1.

D. Derivation of the Income Levels

In order to get a proxy value for the income of each household, I assume the income to be uniformly distributed within the interval of interest. This is a common assumption which is also used for the implementation of the 2011 household income within the microcensus (see the documentation provided by the Statistisches Bundesamt in 2014). Then, the expected household income (i.e. the middle of the income interval) is utilized as a proxy. For the

very top income category, 4085.44 DM (24512.66 EUR) is derived as an upper bound for 1976 (2011). Then, the expected income in the highest income category amounts to 3542.72 DM (21256.33 EUR). For the derivation of the upper bound of the last interval, the upper tail of the 1976 income distribution is assumed to behave like in 2011. Then, the derivation of the upper bound bases on the approach used for the microcensus of 2011 (see Statistisches Bundesamt, 2014). For households with an income larger than the upper bound of 18000, we compute the mean household income, that is 21256.33. The ratio of this mean income and the last known upper interval bound (i.e. 18000) is used to compute a hypothetical upper bound. The ratio is

$$\frac{21256.33}{18000} \approx 1.18.$$

Then, the upper bound as well as the expected household income of the very top income category from 1976 follows immediately.